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LAMINAR FORCED CONVECTION IN RECTANGULAR CHANNELS WITH UNEQUAL HEAT ADDITION ON ADJACENT SIDES

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Abstract—Temperature distributions are determined analytically for fully developed laminar heat transfer in channels with aspect ratios from 1 to ∞. The channel walls are uniformly heated, but the heat flux on the short sides is an arbitrary fraction between 0 and 1 of the heat flux on the broad sides. For all cases, the wall temperatures are compared on the basis that the total heat transferred per unit channel length is maintained at a fixed value. The poor convection due to the low velocities in the corners and along the narrow walls always caused the peak temperatures to occur at the corners. The lowest peak temperatures were found when all the heating took place at only the broad walls rather than when heating was partly distributed to the short sides. This results from the fact that, when four sides are heated, more energy is being supplied to the low velocity corner regions. For heating at only the broad walls, the corner temperature decreases rapidly as the aspect ratio is increased to about 10 and insignificantly thereafter. In the limit of infinite aspect ratio, the wall temperature distribution does not approach a constant as is the case for infinite parallel plates.

NOMENCLATURE

A_n, B_n, Fourier coefficients defined in equations (9) and (10);
a, b, half lengths of short and broad sides, respectively;
C^*, coefficient defined after equation (6);
c_p, specific heat of the fluid;
G, quantity defined after equation (1b);
k, thermal conductivity of the fluid;
p, static pressure;
Q, heat addition per unit channel length;
q, heat addition per unit wall area;
T, temperature;
u, fluid velocity;
$\bar{u}$, mean fluid velocity;
X, dimensionless co-ordinate, x/a;
x, co-ordinate measured along short side from channel center;
Y, dimensionless co-ordinate, y/b;
y, co-ordinate measured along long side from channel center;
z, co-ordinate measured along the axial direction;
β, heat flux ratio, q_S/q_B;
γ, aspect ratio, b/a;
θ, dimensionless temperature, $4kT/Q$;
θ_c, complementary solution given by the sum of θ_1 and θ_2 from equations (9) and (10);
θ_p, particular solution equivalent to $\theta_\pi + \theta^*$ as given by equation (6);
μ, fluid viscosity;
ρ, fluid density.

Subscripts
B, refers to broad side;
b, bulk mean value;
S, refers to short side;
w, value at wall.

INTRODUCTION

RECTANGULAR coolant channels are often employed in heat-exchange devices, particularly in nuclear reactor plate-type fuel assemblies where wide, parallel fuel bearing plates are supported by unfueled side plates. In such assemblies, most of the total heating is produced in the broad, fueled plates with the remainder (usually less than 10 per cent) resulting from gamma heating in the support walls. For example, in [1] 3 per cent of the total heating is generated in the support walls. Cooling is accomplished by passing high velocity fluid through the channels. A factor of importance for proper operation of the reactor is maintaining a satisfactory temperature distribution in the cooling channel walls.

Several papers have treated laminar fully developed heat transfer in rectangular channels with specified heat fluxes around the periphery. In [2] the problem is examined where both uniform and non-uniform heating take place on a large fraction of only the broad walls. As part of the solution in [2], the case of uniform heating over the entire broad walls (with the side walls unheated) was solved numerically for channels having aspect ratios of 10 and 20. In [3] variational methods were utilized to obtain results for aspect ratios of 1 and 10 with uniform heat flux on all four sides, and for an aspect ratio of 10 with uniform heating on the broad sides only. An analytical solution was obtained in [4] for uniform heating on four sides, and results evaluated for aspect ratios of 1, 2, and 4.

It is the purpose of this note to provide the general solution where heating occurs on all four walls for the conditions that the uniform heat flux on the short walls is any fraction between 0 and 1 of the flux on the broad walls. The total heat input per unit channel length is maintained constant, and aspect ratios from 1 to ∞ are considered.

ANALYSIS

The rectangular channel and its co-ordinate system are shown in Fig. 1. Only the fully developed velocity and temperature regions are considered, and the fluid is assumed to have constant properties.

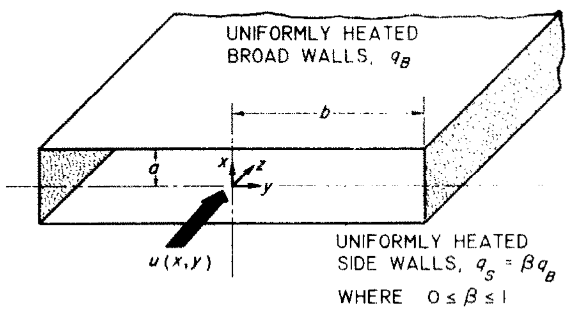

FIG. 1. Co-ordinate system for rectangular channel with different uniform heating on each pair of opposite walls, $Q = 4aq_S + 4bq_B$.

Velocity distribution. For steady laminar flow, the velocity distribution is given in [5] as

$$u = \frac{4a^2}{\pi^3 \mu} \frac{\mathrm{d}p}{\mathrm{d}z} \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n^3} (-1)^{(n+1)/2} \times \left[1 - \frac{\cosh (n\pi y/a)}{\cosh (n\pi b/2a)}\right] \cos \frac{n\pi x}{a} \tag{1a}$$

Equation (1a) was integrated over the cross section, and the double integral divided by the cross-sectional area to give the mean velocity $\bar{u}$ which was used to non-dimensionalize u. In addition the infinite cosine series arising from the 1 in the bracket in equation (1a) is equivalent to a parabola so that the expression can be simplified to the form [6]:

$$\frac{u}{\bar{u}} = G\left\{-\frac{\gamma}{8} + \frac{\gamma X^2}{8} - \frac{4\gamma}{\pi^3} \sum_{n=1,3,5,\ldots}^{\infty} \frac{(-1)^{(n+1)/2}}{n^3 \cosh (n\pi\gamma/2)} \times \left[\cosh \left(\frac{n\pi\gamma}{2} Y\right) \cos \left(\frac{n\pi X}{2}\right)\right]\right\} \tag{1b}$$

where

$$G = \left[-\frac{\gamma}{12} + \frac{16}{\pi^5} \sum_{m=1,3,5,\ldots}^{\infty} \frac{1}{m^5} \tanh \left(\frac{m\pi\gamma}{2}\right)\right]^{-1}$$

Energy equation. The energy equation for the fluid temperature with viscous dissipation neglected, is

$$\rho c_p u \frac{\partial T}{\partial z} = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \tag{2}$$

Under the assumption of constant heat addition per unit channel length, Q, a heat balance on the channel length between the entrance ($z = 0$) and any axial location z shows that the bulk temperature, $T_b(z)$, rises linearly along the axial direction:

$$T_b(z) = T_b(z = 0) + \frac{Qz}{4ab\rho c_p \bar{u}}$$

By definition, the fully developed condition states that $[T(x, y) - T_b]$ does not depend on the axial position z. Hence,

$$\frac{\partial T}{\partial z} = \frac{\partial T_b}{\partial z} = \frac{Q}{4ab\rho c_p \bar{u}}; \quad \frac{\partial^2 T}{\partial z^2} = 0$$

Equation (2) then simplifies to,

$$\frac{u}{\bar{u}}\left(\frac{Q}{4ab}\right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{3}$$

From symmetry, only the first quadrant need be considered, and the boundary conditions are

$$\left.\begin{aligned} 0 \leqslant x \leqslant a: &\quad \frac{\partial T}{\partial y}(x, 0) = 0; \\ &\quad \frac{\partial T}{\partial y}(x, b) = \frac{Q}{4k\,[b/\beta + a]} = \frac{q_S}{k} \\ 0 \leqslant y \leqslant b: &\quad \frac{\partial T}{\partial x}(0, y) = 0; \\ &\quad \frac{\partial T}{\partial x}(a, y) = \frac{Q}{4k\,[b + a\beta]} = \frac{q_B}{k} \end{aligned}\right\} \tag{4}$$

The energy equation (3) is to be solved subject to the boundary conditions (4) using the velocity distribution (1b).

Superposition of solutions. Equation (3) is written in terms of the dimensionless temperature $\theta = 4kT/Q$:

$$\nabla^2\theta \equiv \frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = \frac{1}{ab}\frac{u}{\bar{u}} \tag{3a}$$

This is a non-homogeneous second order partial differential equation, and the difficulty of solution is caused by the complexity of the $u/\bar{u}$ term. By superposition, the solution is expressed as the sum of a particular solution θ_p, and a complementary solution θ_c; i.e. $\theta = \theta_p + \theta_c$ where θ_p satisfies the Poisson equation:

$$\nabla^2\theta_p = \frac{1}{ab}\frac{u}{\bar{u}} \tag{5a}$$

and θ_c satisfies the Laplace equation:

$$\nabla^2\theta_c = 0 \tag{5b}$$

A particular solution can be adapted from the one given in [4] and is written in the dimensionless form:

$$\begin{aligned} \theta_p = \theta_\pi + \theta^* = G\Bigg\{&\frac{X^4}{96} - \frac{\gamma^2 Y^2}{16} - \frac{8}{\pi^5} \\ &\times \sum_{n=1,3,5,\dots}^{\infty} \frac{(-1)^{(n+1)/2}}{n^5 \cosh(n\pi\gamma/2)} \times \Bigg[\cosh\left(\frac{n\pi\gamma}{2}Y\right) \\ &+ \left(\frac{n\pi\gamma}{2}\right) Y \sinh\left(\frac{n\pi\gamma}{2}Y\right)\Bigg] \cos\left(\frac{n\pi X}{2}\right)\Bigg\} \\ &+ C^*(X^2 - \gamma^2 Y^2) \end{aligned} \tag{6}$$

where $C^* = 1/2[\gamma + \beta]$.

The complementary solution is divided into two parts $\theta_c = \theta_1 + \theta_2$ having the boundary conditions

$$\left.\begin{aligned} &\frac{\partial\theta_1}{\partial x}(0, y) = \frac{\partial\theta_1}{\partial y}(x, 0) = \frac{\partial\theta_1}{\partial y}(x, b) = 0 \\ &\frac{\partial\theta_1}{\partial x}(a, y) = \frac{1}{b + a\beta} - \frac{2C^*}{a} - \frac{\partial\theta_\pi}{\partial x}(a, y) \end{aligned}\right\} \tag{7}$$

$$\left.\begin{aligned} &\frac{\partial\theta_2}{\partial x}(0, y) = \frac{\partial\theta_2}{\partial x}(a, y) = \frac{\partial\theta_2}{\partial y}(x, 0) = 0 \\ &\frac{\partial\theta_2}{\partial y}(x, b) = \frac{1}{b/\beta + a} + \frac{2C^* b}{a^2} - \frac{\partial\theta_\pi}{\partial y}(x, b) \end{aligned}\right\} \tag{8}$$

In addition to satisfying these boundary conditions, it is a restriction of the Neumann problem for Laplace's equation that the line integral of the normal derivatives around the boundary be zero, a condition that was used to evaluate C^* from either equations (7) or (8). It is the necessity of satisfying these line integral conditions that required the θ^* function to be introduced.

The solutions for θ_1 and θ_2 were found by using product solutions in conjunction with Fourier series expansions of the boundary conditions. The final solutions for θ_1 and θ_2 in dimensionless forms are:

$$\theta_1 = \sum_{n=1,2,3,\dots}^{\infty} A_n \frac{\cosh(n\pi X/\gamma)}{\sinh(n\pi/\gamma)} \cos(n\pi Y)$$

$$A_n = G\left\{\frac{4\gamma^2(-1)^n}{\pi^6 n} \sum_{m=1,3,5,\dots}^{\infty} \frac{1}{m^3[(m\gamma/2)^2 + n^2]} \times \left[\frac{m\pi\gamma}{2} + \frac{2n^2}{(m\gamma/2)^2 + n^2}\tanh\left(\frac{m\pi\gamma}{2}\right)\right]\right\} \tag{9}$$

$$\theta_2 = \sum_{n=1,2,3,\dots}^{\infty} B_n \frac{\cosh(n\pi\gamma Y)}{\sinh(n\pi\gamma)} \cos(n\pi X)$$

$$B_n = G\left\{\frac{4(-1)^n}{\pi^6 n} \sum_{m=1,3,5,\dots}^{\infty} \frac{1}{m^3[n^2 - (m/2)^2]} \times \left[2\tanh\left(\frac{m\pi\gamma}{2}\right) + \frac{m\pi\gamma}{2}\right]\right\} \tag{10}$$

The analytical forms of θ_1 and θ_2 automatically satisfy the zero derivative conditions in equations (7) and (8). The Fourier coefficients A_n and B_n were evaluated to satisfy the finite derivative conditions $\partial\theta_1(a, y)/\partial x$ and $\partial\theta_2(x, b)/\partial y$, respectively. It is significant to note that the constant terms,

$$\left(\frac{1}{b + a\beta} - \frac{2C^*}{a}\right) \text{ and } \left(\frac{1}{b/\beta + a} + \frac{2C^*b}{a^2}\right),$$

in these boundary conditions do not make a contribution to the A_n and B_n, as they are multiplied by $\cos(n\pi Y)$ and $\cos(n\pi X)$, respectively, and are integrated from 0 to 1 resulting in zero values. Hence, the θ_1 and θ_2 solutions depend only on the θ_π part of the particular solution, and along with θ_π, are independent of β. As a result, it is the β factor in the θ^* function that alone accounts for the unequal heat fluxes on adjacent sides.

Bulk temperature. The solution

$$\theta = \theta_\pi + \theta^* + \theta_1 + \theta_2$$

is substituted into the definition for the bulk temperature:

$$\theta_b = \frac{1}{ab}\int_0^a\int_0^b \frac{u}{\bar{u}}\,\theta\,\mathrm{d}x\,\mathrm{d}y \tag{11}$$

The integrations are carried out, and after considerable algebraic manipulation the bulk temperature is given by

$$\theta_b = \frac{\gamma}{\gamma + \beta}\left(\frac{G}{40} + \frac{G\gamma^2}{72}\right) + \frac{G^2\gamma^3}{576} + \frac{131G^2\gamma}{40320} - \frac{G}{4\pi^3}\sum_{n=1,2,3,\ldots}^{\infty} B_n\frac{(-1)^n}{n^3} + \frac{4G^2\gamma}{\pi^8}\sum_{m=1,3,5,\ldots}^{\infty}\frac{1}{m^8\cosh^2(m\pi\gamma/2)}$$

$$+ G\sum_{n=1,3,5,\ldots}^{\infty}\tanh\left(\frac{n\pi\gamma}{2}\right)\left\{\frac{1}{(n\pi)^5}\left[\frac{\gamma}{\gamma+\beta}\left(\frac{8}{\gamma} - 8\gamma\right) - G\gamma^2 + \frac{G}{6} - \frac{128}{(\gamma+\beta)(n\pi)^2}\right.\right.$$

$$\left.\left. - \frac{16G}{(n\pi)^2} + \frac{72G}{(n\pi)^4}\right]\right\} + \frac{G}{\pi^5}\sum_{m=1,3,5,\ldots}^{\infty}\;\sum_{n=1,2,3,\ldots}^{\infty}\frac{(-1)^n A_n}{m\,[(m/2)^2 + (n/\gamma)^2]^2}\,\frac{\tanh(m\pi\gamma/2)}{\tanh(n\pi/\gamma)}$$

$$\times\frac{2G}{\pi^5}\sum_{m=1,3,5,\ldots}^{\infty}\;\sum_{j=1,2,3,\ldots}^{\infty}\frac{(-1)^j B_j}{m^2[(m/2)^2 - j^2]^2}\left[\frac{m\tanh(m\pi\gamma/2)}{2\tanh j\pi\gamma} - j\right] \tag{12}$$

The final solution is then given as

$$\theta - \theta_b = \theta_\pi + \theta^* + \theta_1 + \theta_2 - \theta_b$$

in which γ and β are the only parameters.

LIMITING CASE OF INFINITE ASPECT RATIO

The solution was examined for the case where $\gamma \to \infty$ resulting in the following limiting analytical expression for the wall temperature distribution:

$$\theta(1, Y) - \theta_b = \left(\frac{96Y^2 - 32}{\pi^5}\right)\left(\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n^5}\right) + \frac{\beta}{2}Y^2 - \frac{\beta}{6} \tag{13}$$

$$\theta(X, 1) - \theta_b = \frac{\beta}{3} + \frac{64}{\pi^5}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n^5} \tag{14}$$

where (13) and (14) agree at the corner $X = 1$, $Y = 1$, and

$$\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n^5} = \frac{31}{32}(1{\cdot}03693)$$

in which the last number is the Riemann zeta function of argument 5. As shown on Fig. 2 this limit yielded results consistent with the general solution evaluated at large γ.

DISCUSSION

Designers of nuclear reactor cooling channels are interested in the temperatures achieved by the walls for a known heat input and a given coolant. The wall temperatures are given here relative to the bulk temperature, and the difference $(T_w - T_b)$ is non-dimensionalized by $Q/4k$. The dimensionless distributions $(T_w - T_b)/(Q/4k)$ were evaluated from the analytical solution and are presented in Figs. 2 to 4. The magnitudes in these figures can be thought of as directly comparable in terms of temperatures if the fluid conductivity k and the total heat input per unit channel length are assumed fixed.

The validity of the analytical solution was verified by comparison with results for a few cases obtained by different methods in [2] and [3] and by a similar analytical technique in [4] for $\beta = 1$. At some places along the wall, the value of $(T_w - T_b)/Q/4k)$ is negative, which means that T_b is larger than T_w. This may seem to contradict the fact that heat is flowing from the wall to the fluid. However, it must be recalled that T_w is a *local value* along the wall, while T_b is an *average value* over the entire cross section.

Because of the complexity of the equations, the influence of the aspect ratio γ and the heating ratio parameter β are not readily explained directly from the analytical solution. For this reason, an attempt is made to present physically plausible explanations for some of the trends in the graphical results. The physical significance of the β parameter should be kept in mind when

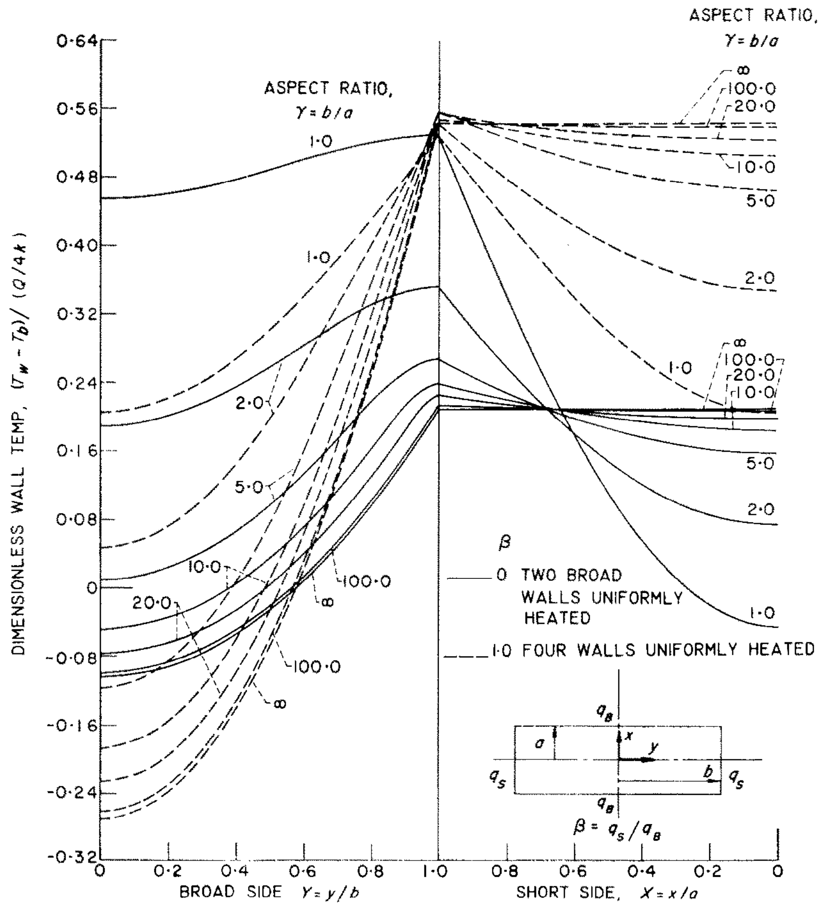

FIG. 2. Wall temperatures of rectangular channels for cases of uniform heat flux on two walls and uniform heat flux on four walls.

interpreting the figures. When $\beta = 0$, all the Q is uniformly transferred from the broad walls only. For a fixed aspect ratio, as β is raised, an increasing portion of the Q is transferred to the fluid from the short sides, so that when $\beta = 1$, all portions of the periphery have the same local heat flux.

Figure 2 provides results for various aspect ratios, for the important cases where either only two walls or all four walls are uniformly heated. Consider first the solid curves, where all of the Q is being dissipated from only the broad walls. The curve for $\gamma = 10$ agreed very well with the results obtained by a variational method in [3], while both the $\gamma = 10$ and 20 curves matched those of [2] which were evaluated by a finite difference technique. As an aid to understanding the curves it is convenient to visualize a set of channels all having the same width $2b$, the same total Q, and with the aspect ratio being increased by diminishing the spacing $2a$. The peak temperatures for all γ's occur in the corners where the low coolant velocities provide for a poorer heat removal than other regions at higher velocities. The increased corner temperatures cause some of the heating in this region to flow into the higher velocity portion of the cross section. For a square duct, $\gamma = 1$, the heat flow paths from the two heated walls toward the region of high velocity fluid are approximately equal for all positions on the heated side, and hence, the temperature along these walls is almost uniform. As γ is increased (by decreasing the spacing, $2a$) the heated wall temperatures decrease because, for narrower ducts, the paths for heat flowing to the region of higher velocities are shortened. These paths are roughly perpendicular to the broad walls except for heat flowing from the corner regions. When γ is large, some of the energy from the corner region must be conducted through a longer path within the fluid in a direction parallel to the broad sides to reach a higher velocity region. Although the region of low velocity fluid occupies proportionately less of the cross-sectional area as γ increases, the width of the heat conduction path parallel to the heated side is also decreased as the duct becomes more narrow. As a result, when $\gamma \to \infty$, the temperature distribution along the heated wall goes to a limit with a maximum in the corner [equations (13) and (14)] rather than to a uniform temperature, as is the case for a duct of infinite parallel plates without bounding side walls. There must always be a spanwise temperature gradient to transport some of the imposed heating away from the low velocity region of the corners and side walls and distribute it to the more rapidly moving fluid.

Now consider the set of dotted curves in Fig. 2 for uniform heating all around the duct periphery. The corner regions now receive heat from two sides and the interesting consequence is that the peak temperature remains essentially constant as γ is increased. The temperature gradients along the broad sides again increase with γ to remove the heat from the corner and side wall regions. Cheng's results for $\gamma = 1$, 2, and 4 were compared to those of this analysis and the agreement was very good for $\gamma = 2$ and 4 (although the $\gamma = 4$ results are not plotted in Fig. 2). Cheng's values for $\gamma = 1$ appear to be in error.

In Fig. 3 is shown the effect of changing β between 0 and 1 for two extremes in aspect ratio, $\gamma = 1$ and 20. As β is increased, the shifting of heat to the narrow wall tends to increase its temperature, while the temperature of the broad wall, tends to decrease. For a square duct, $\gamma = 1$ (dotted lines), the corner temperatures remain fixed because the heating received by the corner region remains constant. The heat removed from one wall that forms the corner is added through the other. For $\gamma = 20$ (solid lines), as β is raised from 0 to 1, only a small amount of heat from all along the broad walls needs to be shifted to the short walls to provide the same uniform heat flux at the short wall. Hence, more energy is concentrated in the region of poor convection and produces an increase of both the spanwise temperature gradient and the corner temperature as β is increased, a result which was found to hold true for all rectangular ducts ($\gamma > 1$). This provides the *important conclusion* that it is better to transfer all the heat through the broad walls of rectangular channels than to distribute it around the entire periphery (under the restrictions that the walls are non-conducting and that the heating extends all the way into the corners). A similar conclusion was also indicated in [3] for an aspect

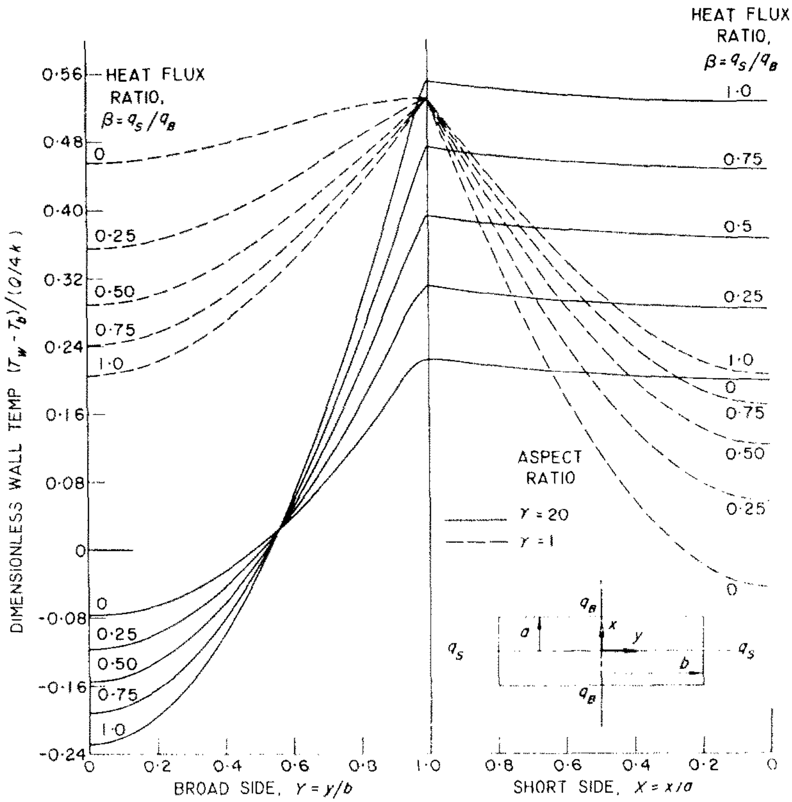

FIG. 3. Wall temperatures for various ratios, β, of the heat flux on the short walls to that on the broad walls in channels with aspect ratios $\gamma = 1$ and 20.

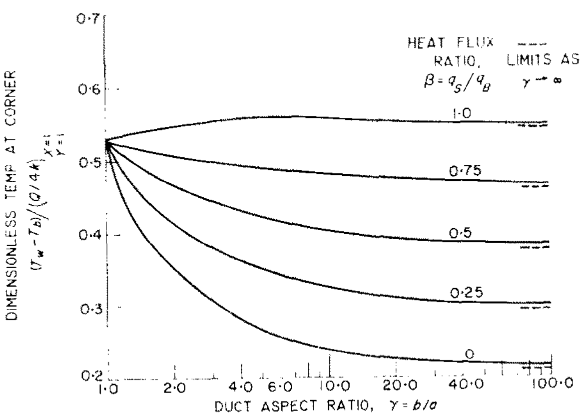

FIG. 4. Dependence of temperature at duct corner on aspect ratio, γ, for various values of heating distribution parameter, β.

ratio of 10 by comparing the two cases given there for $\beta = 1$ and 0.

In Fig. 4 are shown the peak temperatures as a function of the aspect ratio for various β values. The largest reduction in the corner temperatures is achieved when all the heat is transferred through the broad walls only ($\beta = 0$) and when the aspect ratio is increased to about 10. Beyond $\gamma \approx 10$, only a small reduction occurs, so that for rectangular cooling channels in nuclear reactors where $\beta \approx 0{\cdot}03$, the optimum aspect ratio appears to be about 10 to 20. If the side wall heating is increased ($\beta > 0$), the corner temperatures do not drop off as rapidly with larger γ as for the case where $\beta = 0$. When the heat flux is very nearly uniform over all walls ($0{\cdot}75 < \beta < 1$), the peak temperatures are almost constant for all aspect ratios. Hence, it is concluded that for many nuclear reactors that utilize rectangular channels, it would be a disadvantage to load the narrow side walls even if it were possible. The advantage gained by the increased heat-transfer area would be offset by the additional heating imposed near the corner resulting in higher corner temperatures and higher maximum to minimum wall temperature differences.

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Résumé—On détermine analytiquement les distributions de température pour un transport de chaleur laminaire entièrement développé dans des canaux avec des allongements allant de 1 à l'infini. Les parois des canaux sont chauffées uniformément, mais le flux de chaleur sur les faces étroites est une fonction arbitraire comprise entre 0 et 1 du flux de chaleur sur les faces larges. Pour tous les cas, les températures de paroi sont comparées sur la base que la chaleur totale transmise par unité de longueur du canal est maintenue à une valeur fixe. La faible convection due aux vitesses faibles dan les coins et le long des parois étroites faisait toujours que les températures maximales se présentaient aux coins. On a trouvé les températures maximales les plus basses lorsque tout l'échauffement se produisait seulement sur les parois larges plutôt que lorsqu'il était distribué en partie sur les faces étroites. Ceci résulte du fait que, lorsque les quatre côtés sont chauffés, plus d'énergie est fournie aux régions des coins à faible vitesse. Pour l'échauffement seulement aux parois larges, la température des coins diminue rapidement lorsque l'allongement augmente jusqu'à 10 environ et d'une façon insignifiante après. A la limite d'un allongement infini, la distribution de température pariétale ne tend pas vers une constante, comme c'est le cas pour des plaques planes infinies.

Zusammenfassung—Für voll ausgebildeten laminaren Wärmeübergang in Kanälen mit Längenverhältnissen von 1 bis ∞ werden Temperaturverteilungen analytisch bestimmt. Die Kanalwände sind gleichmässig beheizt, doch ist der Wärmestrom von den schmalen Seitenflächen ein willkürlicher Bruchteil zwischen 0 und 1 des Wärmestroms von den breiten Deck- und Bodenflächen. In allen Fällen werden die Wandtemperaturen so verglichen, dass die, pro Längeneinheit des Kanals übertragene Wärmemenge als konstant angenommen wird. Die wegen der kleinen Geschwindigkeiten in den Ecken und entlang der Schmalseiten geringe Konvektion lässt stets die Temperaturspitzen an den Ecken entstehen. Die kleinsten Temperaturspitzen ergeben sich, wenn die Beheizung nur von den breiten Wänden her erfolgt und nicht noch teilweise auf die Schmalseiten verteilt ist. Das beruht darauf, dass bei vier beheizten Seiten, den Eckbereichen mit kleiner Geschwindigkeit mehr Energie zugeführt wird. Bei Beheizung nur von den breiten Flächen her verkleinern sich die Ecktemperaturen bei Steigerung auf das Längenverhältnis 10 sehr rasch, darüber hinaus nur noch unbedeutend. Für den Grenzwert unendlichen Längenverhältnisses nähert sich die Wandtemperaturverteilung keinem konstanten Wert, im Gegensatz zu unendlichen parallelen Platten.

Аннотация—Аналитически определены распределения температуры для полностью развитого ламинарного теплообмена в каналах с отношением b/a от 1 до ∞. Стенки канала нагревались равномерно, но плотность теплового потока с узкой стороны канала составляла некоторую произвольную долю от плотности теплового потока на широкой стороне изменявшуюся от 0 до 1. Для всех случаев температуры стенки сравнивались на той основе, что общее количество тепла, переносимого на единицу длины канала, поддерживалось постоянным. Плохая конвекция из-за малых скоростей в углах и около узких стенок всегда вызывала пики температуры в углах. Меньшие пики температуры были найдены в случае, когда нагревание происходило только на широких стенках по сравнению со случаем, когда часть тепла подводилась через узкие стенки. Это вытекает из того факта, что при нагреве четырех стенок, подводилось больше энергии к углам, где скорости невелики. При нагреве только широких стенок, температуры ь углах быстро уменьшается, с увеличением отношения b/a примерно до 10, а далее изменяется незначительно. В пределе при бесконечном отношении b/a распределение температуры стенки не достигает постоянной величины, как в случае бесконечных параллельных пластин.